NEUTRON STARQUAKES AND THE NATURE OF GAMMA-RAY BURSTS

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ABSTRACT

We investigate the possibility that gamma-ray bursts originate from quakes deep in the solid crust of a neutron star. Seismic waves are radiated if shear stress is relieved by brittle fracture. However they cannot propagate directly to the surface but are temporarily trapped below a reflecting layer. The shaking of the stellar surface couples the seismic waves to Alfvén waves which propagate out into the magnetosphere. The crust-magnetosphere transmission coefficient strongly increases with wave frequency and magnetic field strength. Alfvén wave luminosities sufficient to power galactic gamma-ray bursts are possible if magnetic fields $\gtrsim 10^{11}$G cover at least part of the stellar surface. As the Alfvén waves propagate out into the low density magnetosphere, they become increasingly charge starved, thereby accelerating particles to relativistic energies.

Keywords: gamma-ray bursts, neutron stars, wave motions

1. INTRODUCTION

Fifteen years after their discovery (Ref. 1), gamma-ray bursts remain a major unsolved problem of high energy astrophysics. The bursts lack counterparts at other wavelengths, so the source objects are still unidentified. Neutron stars have long been viewed as being the most plausible sources. Individual bursts usually last for a few seconds, with rise times as short as several milliseconds (Ref. 2). The relative lack of low-energy X-rays, $L_x/L_\gamma \simeq 0.02$, is an important clue to understanding the nature of the gamma-ray emission region. This *X-ray paucity constraint* (Ref. 3) is unlikely to be satisfied in regions of high density or near the neutron star surface, where substantial thermal reprocessing of gamma-rays into X-rays would occur. It suggests emission from a region of low-density plasma well out in the magnetosphere.

The basic idea of starquake models for gamma-ray bursts is quite simple (Refs. 4-7). Elastic energy released in a crustquake excites oscillations of the magnetic field frozen in the surface, and the induced electric field accelerates high-energy particles which in turn radiate gamma-rays. This work is devoted to assessing the merits of starquakes as the root cause of gamma-ray bursts. A detailed discussion of this model can be found in Ref. 8.

2. QUAKE ENERGETICS

The energy released in a typical gamma-ray burst, assuming isotropic emission, is

$$E_b \approx \frac{10^{38}}{\eta}\left(\frac{F}{10^{-6}\mathrm{ergs\,cm^{-2}}}\right)\left(\frac{D}{1\mathrm{kpc}}\right)^2 \mathrm{ergs}, \quad (1)$$

where η specifies the fraction of energy released which is converted into gamma-rays, F is the observed fluence, and D is the distance.

We model the crust as plane parallel, chemically homogeneous, and subject to a constant Newtonian gravitational acceleration, $\mathbf{g} = -g\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a unit vector in the upward direction. In numerical expressions we take $g = 10^{14}\,\mathrm{cm\,s^{-2}}$. Above neutron drip, the pressure is due to degenerate electrons. Integrating the equation of hydrostatic equilibrium, we obtain the density profile

$$\rho = \frac{(\mu_e m_u)^{5/2}}{3\pi^2\hbar^3}\left(\frac{g^2\mu_e m_u}{c^2}z^2 + 2gm_e|z|\right)^{3/2}, \quad (2)$$

where m_u is the atomic mass unit, μ_e is the mean molecular weight per electron, and all other symbols have their usual meanings. We adopt an intermediate value of $\mu_e = 2.5$ when presenting numerical results.

The ions in the solid crust are arranged in a Coulomb lattice whose shear modulus is given by

$$\mu = 0.295 Z^2 e^2 n_i^{4/3}, \quad (3)$$

where n_i is the ion number density and Z is the atomic number. An average value of $Z = 32$ is adopted here. The elastic energy released in a quake may be expressed as

$$E_Q \sim \mu(z_Q)\epsilon_{yield}^2 d^3, \quad (4)$$

where ϵ_{yield} is the yield strain at which the crust cracks, d^2 is the area of the fault plane, and z_Q is the depth. Equations (3) and (4) together require:

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$$d \sim 3 \times 10^4 \left(\frac{E_Q}{10^{38}\mathrm{ergs}}\right)^{1/3} \left(\frac{\epsilon_{yield}}{10^{-2}}\right)^{-2/3} \mathrm{cm}, \quad (5)$$

where, for convenience, the quake is assumed to take place at neutron drip.

The characteristic frequency, ν_0, of elastic waves emitted by the quake is simply the speed at which the fracture propagates divided by d. Since cracks typically propagate at a significant fraction of the shear wave speed, $v_s = (\mu/\rho)^{1/2}$, we find

$$\nu_0 \sim 10^4 \left(\frac{E_Q}{10^{38}\mathrm{ergs}}\right)^{-1/3} \left(\frac{\epsilon_{yield}}{10^{-2}}\right)^{2/3} \mathrm{Hz}. \quad (6)$$

The seismic energy density is partitioned into shear and compressional waves in proportion to the inverse sixth power of the ratio of their propagation speeds. Most of the energy is emitted in shear waves since their speed is typically a few times smaller than that of compressional waves.

3. MAGNETOELASTODYNAMICS

The magnetic energy density dominates the rigidity in the outer layers of the crust of magnetic neutron stars and exceeds the rest mass energy density in the magnetosphere. To include the effects of the magnetic field, $\boldsymbol{B}$, on wave propagation, we add the Maxwell stress to the equations of elastodynamics. For simplicity, we consider the unperturbed magnetic field to be uniform and we treat the surface layers as solid because, with our scalings, the magnetic field completely dominates the stress in the ocean provided the surface temperature is $\lesssim 3 \times 10^5$K. This is probably true for old neutron stars.

Linearizing the equation of motion and the continuity equation about a static equilibrium, we obtain

$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \boldsymbol{\nabla} \cdot \delta\sigma + \frac{1}{c}\delta \boldsymbol{j} \times \boldsymbol{B} + \delta\rho \boldsymbol{g} - \boldsymbol{\nabla}\delta p, \quad (7)$$

$$\delta\rho = -\boldsymbol{\nabla}\cdot(\rho\boldsymbol{\xi}), \quad (8)$$

respectively. Here, δ denotes an Eulerian perturbation, $\boldsymbol{\xi}$ is the displacement of an element of material from its equilibrium position, and $\delta\boldsymbol{j}$ is the perturbed current density. The pressure due to the degenerate electrons, p, has been written separately from the elastic stress associated with the deformation of the Coulomb lattice, σ. The components of the perturbed elastic stress tensor are related to the components of the gradient of $\boldsymbol{\xi}$ by

$$\delta\sigma_{ij} = \left(\kappa - \frac{2\mu}{3}\right)\delta_{ij} \boldsymbol{\nabla} \cdot \boldsymbol{\xi} + \mu \left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i}\right), \quad (9)$$

where κ is the bulk modulus. In writing equation (9), we have neglected the extra terms which arise if there is a static elastic stress associated with the equilibrium state. It is easy to show that these terms are of order $\epsilon_{yield} \ll 1$ relative to those retained.

The crust is effectively a perfect electrical conductor at seismic frequencies, so the perturbed electric field is given by

$$\delta\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{\xi}}{\partial t} \times \boldsymbol{B}. \quad (10)$$

Equation (10) combined with Maxwell's equations,

$$\boldsymbol{\nabla} \times \delta\boldsymbol{E} = -\frac{1}{c}\frac{\partial \delta\boldsymbol{B}}{\partial t} \quad (11)$$

$$\boldsymbol{\nabla} \times \delta\boldsymbol{B} = \frac{4\pi}{c}\delta\boldsymbol{j} + \frac{1}{c}\frac{\partial \delta\boldsymbol{E}}{\partial t}, \quad (12)$$

may be used to relate $\delta\boldsymbol{j}$ to $\boldsymbol{\xi}$.

The transmission coefficient for the energy flux between the crust and the magnetosphere should not be significantly affected by the polarization and propagation angle of the incident shear wave. We consider then the simplest possible case, a vertically propagating shear wave polarized such that $\boldsymbol{\xi} \propto \boldsymbol{z} \times \boldsymbol{B}$. For this special case $\boldsymbol{\nabla} \cdot \boldsymbol{\xi}$, $\delta\rho$, and δp all vanish. Next, we take a harmonic time dependence, $\exp(-i\omega t)$, for the perturbation variables and combine equations (7) - (12) to derive the wave equation,

$$\frac{d}{dz}\left(\tilde{\mu}\frac{d\xi}{dz}\right) + \tilde{\rho}\omega^2 \xi = 0, \quad (13)$$

where $\tilde{\mu}$ and $\tilde{\rho}$ are the effective shear modulus and density:

$$\tilde{\mu} \equiv \left(\mu + \frac{(B\cos\alpha)^2}{4\pi}\right), \quad (14)$$

$$\tilde{\rho} \equiv \left(\rho + \frac{B^2}{4\pi c^2}\right). \quad (15)$$

Here $\cos\alpha \equiv B_z/B$. Equations (2) and (3) are used to obtain the dependence of μ and ρ on depth. The magnetoelastic wave speed, $\tilde{v}_s$, is equal to $(\tilde{\mu}/\tilde{\rho})^{1/2}$. In the magnetosphere, $z > 0$, $\tilde{\mu} = (B\cos\alpha)^2/(4\pi)$, $\tilde{\rho} = B^2/(4\pi c^2)$, and the wave equation reduces to that for relativistic Alfvén waves,

$$\frac{d^2\xi}{dz^2} + \sec^2\alpha \frac{\omega^2}{c^2}\xi = 0. \quad (16)$$

The transmission coefficient, T, is defined as the ratio of the transmitted to the incident energy flux. For a homogeneous crust, it is easy to show that the transmission coefficient for a vertically propagating shear wave is given by the familiar formula

$$T = \frac{4 Z_C Z_M}{(Z_C + Z_M)^2}, \quad (17)$$

where Z_C and Z_M are the impedances of the crust and magnetosphere. Each impedance is the product of the relevant propagation speed and effective density. Thus,

$$Z_C = (\tilde{\rho}\tilde{\mu})^{1/2} \text{ and } Z_M = \frac{B^2\cos\alpha}{4\pi c}. \quad (18)$$

To determine the reflection and transmission amplitudes for an inhomogeneous crust, we perform two numerical integrations of equation (13) from $z \ll 0$ to $z > 0$ starting from different lower boundary conditions. Then we choose the appropriate linear combination of the two boundary conditions such that the solution satisfies the radiation condition, $\xi' = i\omega \sec\alpha\, \xi/c$, for $z > 0$. The curve in figure 1 displays T as a function of frequency for various vertical magnetic field strengths. Only a small fraction T of the incident wave flux leaks into the magnetosphere.

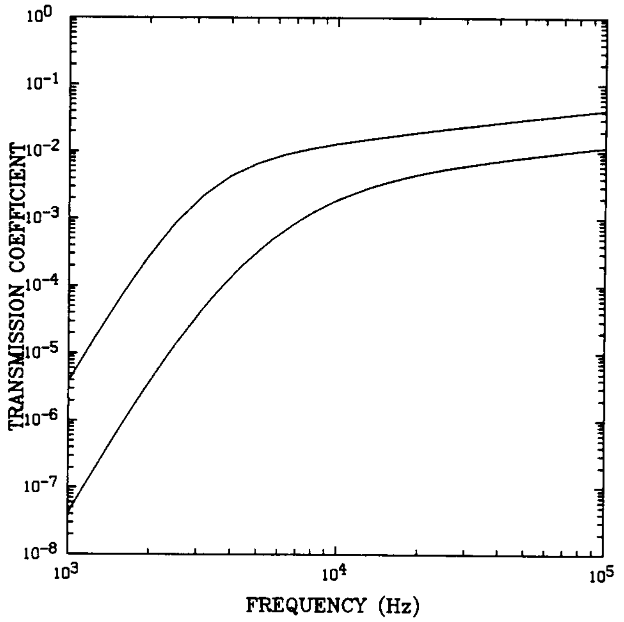

Figure 1: The transmission coefficient as a function of frequency for a vertically propagating wave. The upper and lower pairs of curves are for 10^{12} and 10^{11}G fields respectively.

The fate of the reflected energy is clearly of interest. The angular momentum barrier (or equivalently, the $1/r$ dependence of the horizontal wave number in a spherical star) and an increase in wave speed with depth both refract waves upward. Shear waves will not propagate in the fluid interior below the crust. Those which reach this region may be reflected if the rigidity drops sufficiently abruptly to zero at the inner boundary of the crust. Otherwise, as the waves slow down their radial wave vectors and amplitudes will increase, and their energy will ultimately be dissipated as heat.

The waves which do return will bounce many times off the surface and will spread throughout the crust. The characteristic storage time for wave energy in the crust, $\tau(\nu)$, is twice the time it takes the waves to cross the crust, $2t_c$, divided by $T(\nu)$. For vertically propagating waves,

$$\tau(\nu) \sim -\frac{2}{T(\nu)} \int_0^{z_Q} \frac{dz}{v_s} \sim \frac{5 \times 10^{-6} |z_Q|^{1/2}}{T(\nu)}\ \mathrm{s}. \qquad (19)$$

With the usual scalings, $\tau \sim 1$s at $\nu_0 = 10^4$Hz.

Our analysis of the propagation of shear waves has neglected damping. Theoretical calculations of the degenerate electron viscosity in a solid neutron star crust (Ref. 9) included electron scattering by phonons, impurity ions, other electrons, and free neutrons. We neglect chemical impurities because of the large uncertainty in their concentration and type. At neutron drip, the damping time is of order 4×10^3s for 10^4Hz waves, the dominant scattering mechanism being due to phonons. This is probably an underestimate because of the likely presence of other sources of electron scattering, especially lattice imperfections. Even so, it is still much longer than the typical timescale for a gamma-ray burst.

Up to this point we have considered how a vertically propagating shear wave polarized orthogonal to the ambient magnetic field couples to an Alfvén wave. Since the magnetosphere can also support fast magnetosonic waves, it is natural to inquire whether more general seismic disturbances couple to them. Now, the dispersion relation for fast magnetosonic waves is isotropic and reads

$$\omega^2 = (ck)^2, \qquad (20)$$

which differs from the anisotropic dispersion relation for Alfvén waves,

$$\omega^2 = (ck\cos\psi)^2. \qquad (21)$$

Here, k is the magnitude of the wave vector, and ψ is the angle between $\boldsymbol{k}$ and $\boldsymbol{B}$. For fast magnetosonic waves and Alfvén waves, Snell's law reads

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c}{\tilde{v}_s}, \qquad (22)$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c\cos\psi}{\tilde{v}_s}. \qquad (23)$$

Here θ_i and θ_t denote the angles of incidence and transmission. At the reflecting layer, $\tilde{v}_s \ll c$, and thus only waves having very small angles of incidence can couple to fast magnetosonic waves. For Alfvén waves the only restriction is that the transmitted wave vector be almost orthogonal to $\boldsymbol{B}$.

The displacement amplitude of a *propagating* shear wave increases with height in the crust as a consequence of the conservation of energy flux. Because the strain amplitude, ϵ, is proportional to the gradient of the displacement, and the magnitude of the wave vector, $k = \omega/\tilde{v}_s$, also increases with height, ϵ reaches a maximum near the surface. The following argument relates the maximum strain amplitude to the Alfvén wave luminosity.

We assume that the entire neutron star surface radiates Alfvén waves. Then, the fractional perturbation of the surface magnetic field associated with luminosity L is

$$\frac{\delta B}{B} \sim 6 \times 10^{-4} \left(\frac{L}{10^{38}\mathrm{ergs\,s^{-1}}}\right)^{1/2} \left(\frac{B}{10^{11}\mathrm{G}}\right)^{-1}. \qquad (24)$$

In terms of the surface displacement amplitude, equations (10) and (11) imply

$$\frac{\delta B}{B} \sim \frac{\omega}{c}\xi. \qquad (25)$$

Thus, we estimate the total surface strain to be:

$$\epsilon \sim 0.2 \left(\frac{v_s}{10^8 \mathrm{cm\,s^{-1}}}\right)^{-1} \left(\frac{L}{10^{38} \mathrm{ergs\,s^{-1}}}\right)^{1/2} \left(\frac{B}{10^{11}\mathrm{G}}\right)^{-1}. \tag{26}$$

With the chosen scalings, the maximum strain is dangerously close to unity. A *dynamic* yield strain as large as 0.1 is not unusual. Thus, we cannot predict whether shear waves transfer their energy to Alfvén waves and/or crumble the crust, generating heat. However, it is clear that the neutron starquake model is not viable if bursts are significantly more distant, and consequently more luminous, than estimated above.

4. ALFVÉN WAVES IN THE MAGNETOSPHERE

We have already discussed the magnetic field perturbations required for Alfvén waves to carry the luminosity of a typical gamma-ray burst. Because Alfvén waves transport energy without loss along the equilibrium field lines, their relative amplitudes vary according to

$$\frac{\delta B}{B} \propto B^{-1/2}. \tag{27}$$

Equations (24) and (27) indicate that the Alfvén waves are likely to become nonlinear not very far from the star.

Up to this point, we have been making the implicit assumption that the plasma density in the magnetosphere is high enough so that the MHD limit applies to Alfvén wave propagation. We shall now show that this is a questionable assumption.

The corotation charge number density (Ref. 10),

$$n_{cr} \simeq \frac{B}{Pce} \simeq 7 \times 10^9 \left(\frac{B}{10^{11}\mathrm{G}}\right) \left(\frac{P}{1\mathrm{s}}\right)^{-1} \mathrm{cm}^{-3}, \tag{28}$$

sets a lower limit on the magnetospheric plasma density, n. Here, P is the rotational period of the neutron star. Much higher densities may exist if accretion is occurring, or if an energetic process expells plasma from the stellar atmosphere.

For low plasma densities a nonlinearity may occur even at relative field amplitudes $\delta B/B \ll 1$. Because the wave vector is nearly orthogonal to the magnetic field near the surface, equation (10) implies that the wave vector is nearly parallel to $\delta \boldsymbol{E}$. Therefore, the displacement current and $\boldsymbol{\nabla} \times \delta \boldsymbol{B}$ are nearly orthogonal. From equation (12), we see that a substantial physical current is necessary to support the wave. If the plasma density is so low that the required drift velocities exceed c, the wave is *charge starved*. This occurs where

$$\frac{\delta B}{B} \sim \frac{2nec}{\nu_0 B} \sim 10^{-4} \left(\frac{n}{n_{cr}}\right) \left(\frac{P}{1\,\mathrm{s}}\right)^{-1}. \tag{29}$$

Based on the values of $\delta B/B$ given by equation (24), the Alfvén waves are expected to be charge starved close to the stellar surface unless $n \gg n_{cr}$. The fate of such waves requires further investigation.

Once the Alfvén waves go nonlinear, a substantial part of the wave energy is probably transferred to the electrons. An upper limit to the Lorentz factors is obtained by balancing the electrostatic acceleration with curvature radiation reaction. We find

$$\gamma \sim 10^7 \left(\frac{\delta B/B}{10^{-4}}\right)^{1/4} \left(\frac{B}{10^{11}\mathrm{G}}\right)^{1/4} \left(\frac{R}{10^6\,\mathrm{cm}}\right)^{1/2}. \tag{30}$$

5. CONCLUSIONS

Starquakes have the following virtues. They can easily provide the energy necessary to power a single burst. They release energy in the low entropy form of seismic waves which are transformed into Alfvén waves on timescales characteristic of observed gamma-ray bursts. The Alfvén waves may become nonlinear far from the the stellar surface and accelerate the ambient plasma to energies that are limited by radiation reaction losses due to the emission of gamma-rays. Thus, it is possible to imagine an efficient conversion of elastic energy to gamma-ray energy.

Of course, the above scenario is replete with uncertainty. It is quite possible that plastic flow, and not brittle fracture, is the manner in which stress is relieved in neutron star crusts. The source of the free energy necessary to replenish the crustal stress in old neutron stars is not identified. The conversion of seismic waves into Alfvén waves, on the appropriate timescales and without the production of excessive crustal strains, requires that magnetic fields $\gtrsim 10^{11}$G cover at least part of the stellar surface. The propagation of Alfvén waves in low density, neutron star magnetospheres is poorly understood. Their behavior and the manner in which they accelerate particles probably depends upon the degree to which the neutral plasma density exceeds the corotation change density.

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